INSIGHT 1 (**Problem of the parsimonious description**)

Identifying a model having few parameters and describing a large class of stochastic processes. □

INSIGHT 2 (**K-factor GARMA model**)

[THE (?) parsimonious model encompassing a LARGE class of random processes (Fractionally differenced, AutoRegressive, Moving Average, etc), ...difficult to beat, without too much complicating the model!]

A K-factor GARMA (Gegenbauer AutoRegressive Moving Average) model satisfies (time domain)

$$\Phi(B)\prod_{k=1}^{K}\left(1-2\psi_k B+B^2\right)^{\delta_i} X(t)=\Theta(B)Z(t), \tag{1}$$

where $Z(t)$ is a zero-mean white noise with variance σ^2, functions Θ, Φ are defined by $\Theta(B)=I-\sum_{p=1}^{P}\theta_p B^p$, $\Phi(B)=I-\sum_{q=1}^{Q}\phi_q B^q$, B is the backshift operator*: $BX(t)=X(t-1)$ and I represents the identity operator. Parameters $(\phi_\ell)_\ell$ model the contribution of autoregressive terms and parameters $(\theta_\ell)_\ell$ correspond to moving average contributions.* □

INSIGHT 3 (**K-factor GARMA spectrum**)

3) *The spectrum γ of a K-factor GARMA random process is of the form*

$$\gamma(\omega)=\Lambda(\omega)\times\Psi(\omega),\ \textit{with}\quad \Lambda(\omega)=\frac{\Theta(e^{-i\omega})}{\Phi(e^{-i\omega})}=\frac{\left|1-\sum_{p=1}^{P}\theta_p e^{-ip\omega}\right|^2}{\left|1-\sum_{q=1}^{Q}\phi_q e^{-iq\omega}\right|^2}\ \textit{and}\quad \Psi(\omega)=\prod_{k=1}^{K}\frac{\sigma^2}{\left\{2|\cos\omega-\psi_k|\right\}^{2\delta_k}}, \tag{2}$$

where function Λ provides the ARMA spectral contribution and Ψ denotes the following "fractional K-factor" function: □

INSIGHT 4 (**Poles of K-factor GARMA spectrum**)

A K-factor GARMA spectrum can admit up to $Q+K+1$ poles.

When some poles of γ are very close to each other, estimating these poles is difficult because their lobes/peaks tend to overlap from standard Fourier based spectrum estimators. □

♠ **The Discrete Wavelet Packet Transform (DWPT) is relevant for discriminating the poles of γ with high accuracy [these poles are associated with singular DWPT paths with unbounded set of variances], see [1].**

[1] A. M. Atto and Y. Berthoumieu, "Wavelet packets of nonstationary random processes: Contributing factors for stationarity and decorrelation," *IEEE Transactions on Information Theory*, vol. 58, no. 1, Jan. 2012. Available: http://dx.doi.org/10.1109/TIT.2011.2167496