Wavelet Packet Spectrum [2-D version for this digest]

- Consider a "sympathetic"[1] zero-mean second order random field X.
- Consider the 2-Dimensional (2-D) Discrete Wavelet Packet Transform (DWPT) associated with a decomposition level j and a pair of frequency indices $(n_1, n_2) \triangleq n$.
- Use the Shannon wavelet filters to compute the DWPT coefficients of X.
- Then, the autocorrelation functions of these coefficients have the form

$$R^S_{j,n}[m_1, m_2] = \frac{2^{2j}}{\pi^2} \times \int_{\left[\frac{G(n_1)\pi}{2^j}, \frac{(G(n_1)+1)\pi}{2^j}\right] \times \left[\frac{G(n_2)\pi}{2^j}, \frac{(G(n_2)+1)\pi}{2^j}\right]} \gamma(\omega_1, \omega_2) \cos(2^j m_1 \omega_1) \cos(2^j m_2 \omega_2)\, d\omega_1\, d\omega_2, \quad (1)$$

  where γ is the spectrum[2] of the X (see [1] for details).
- ★ Consider a continuity point $\omega = (\omega_1, \omega_2)$ of γ. Then, we have

$$\gamma(\omega) = \lim_{j \to +\infty} R^S_{j,n_{\mathcal{P}}(j)}[0, 0], \quad (2)$$

  where frequency indices $(n_{\mathcal{P}}(j))_{j \geqslant 0}$ issue from a particular DWPT path $\mathcal{P}$ guarantying

$$\omega = \lim_{j \to +\infty} \frac{G(n_{\mathcal{P}}(j))\,\pi}{2^j} \quad (3)$$

  and function G relates to the inverse of the Gray code permutation (see [3], [1] for details).
- Since the random field is assumed to have zero-mean (remove the mean before decomposing, in practice), then $R^S_{j,n}[0, 0] = \mathrm{var}[c_{j,n}]$ is the variance of the DWPT (j, n)-subband.
- Spectrum γ can thus be estimated by computing and ordering conveniently, the variances of Shannon DWPT coefficients located at sufficiently large decomposition levels.
- Shannon DWPT decomposition is not an easy task (try, and tell me if so ...): in practice, use a Daubechies wavelet with large order (for approximating the "ideal" Shannon case).

■ *Refer to [1], [2], [4], [5] in order to go beyond this brief presentation.* ■

References

[1] A. M. Atto, Y. Berthoumieu, and P. Bolon, "2-dimensional wavelet packet spectrum for texture analysis," *IEEE Transactions on Image Processing*, Forthcoming 2012.

[2] A. M. Atto, D. Pastor, and G. Mercier, "Wavelet packets of fractional brownian motion: Asymptotic analysis and spectrum estimation," *IEEE Transactions on Information Theory*, vol. 56, no. 9, Sep. 2010.

[3] M. V. Wickerhauser, *Adapted Wavelet Analysis from Theory to Software.* AK Peters, 1994.

[4] A. M. Atto, D. Pastor, and A. Isar, "On the statistical decorrelation of the wavelet packet coefficients of a band-limited wide-sense stationary random process," *Signal Processing*, Elsevier, vol. 87, no. 10, pp. 2320 – 2335, Oct. 2007.

[5] D. Pastor and R. Gay, "Décomposition d'un processus stationnaire du second ordre: Propriétés statistiques d'ordre 2 des coefficients d'ondelettes et localisation frequentielle des paquets d'ondelettes," *Traitement du Signal*, vol. 12, no. 5, 1995.

[1] See [1], [2] for limitations. These limitations are mainly due to the definition and integrability of DWPT coefficients. In practice, when dealing with [necessarily finite] discrete samples of an observed field, these limitations have no or little effect.

[2] Function γ is the standard Power Spectral Density (Fourier transform of autocorrelation) for a wide sense stationary field.

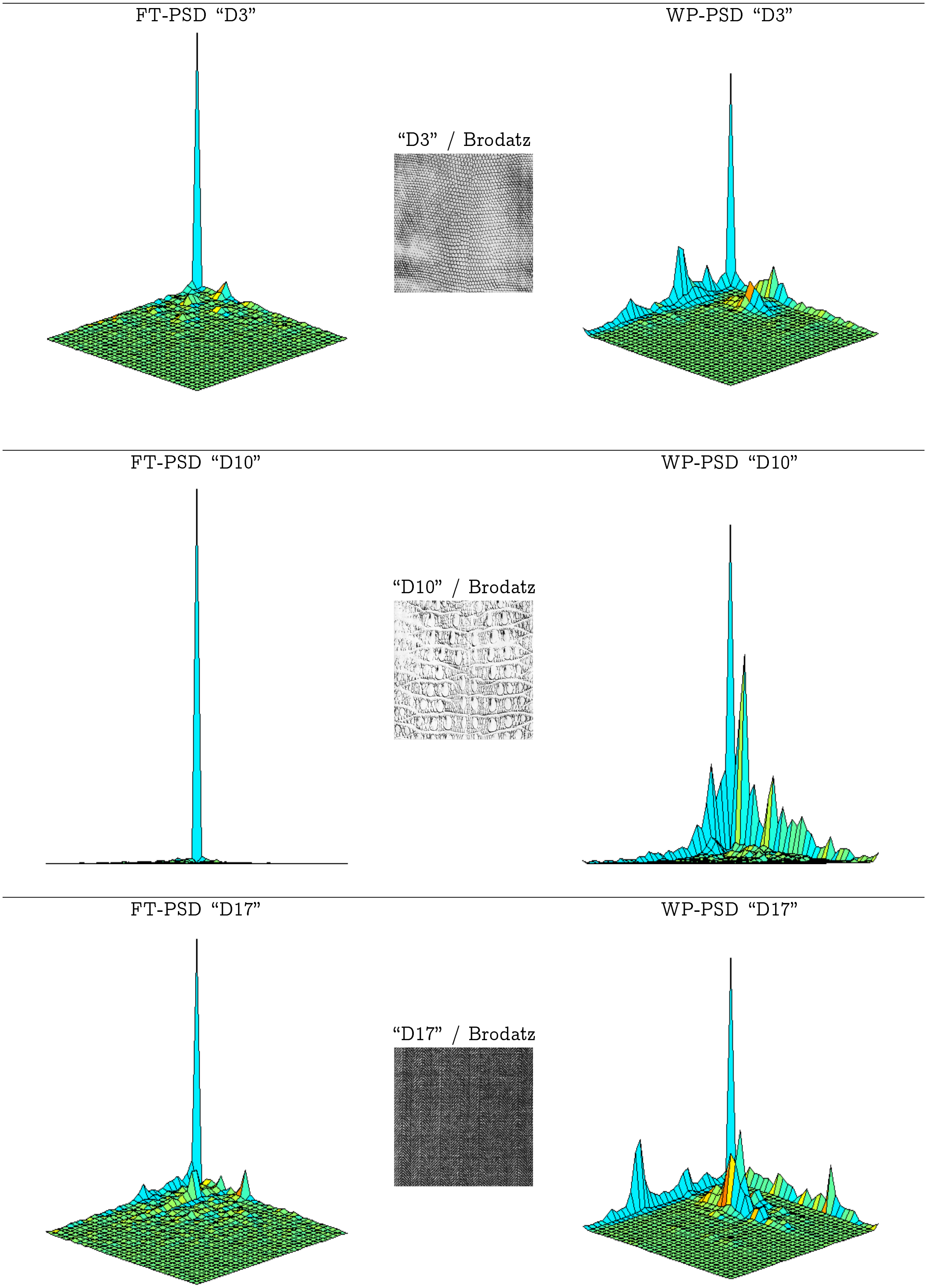

Fig. 1. Texture images and their spectra $\widehat{\gamma}$ computed by using Fourier and wavelet packets. Abscissa of the spectra images consist of a regular grid over $[0, \pi/2] \times [0, \pi/2]$.

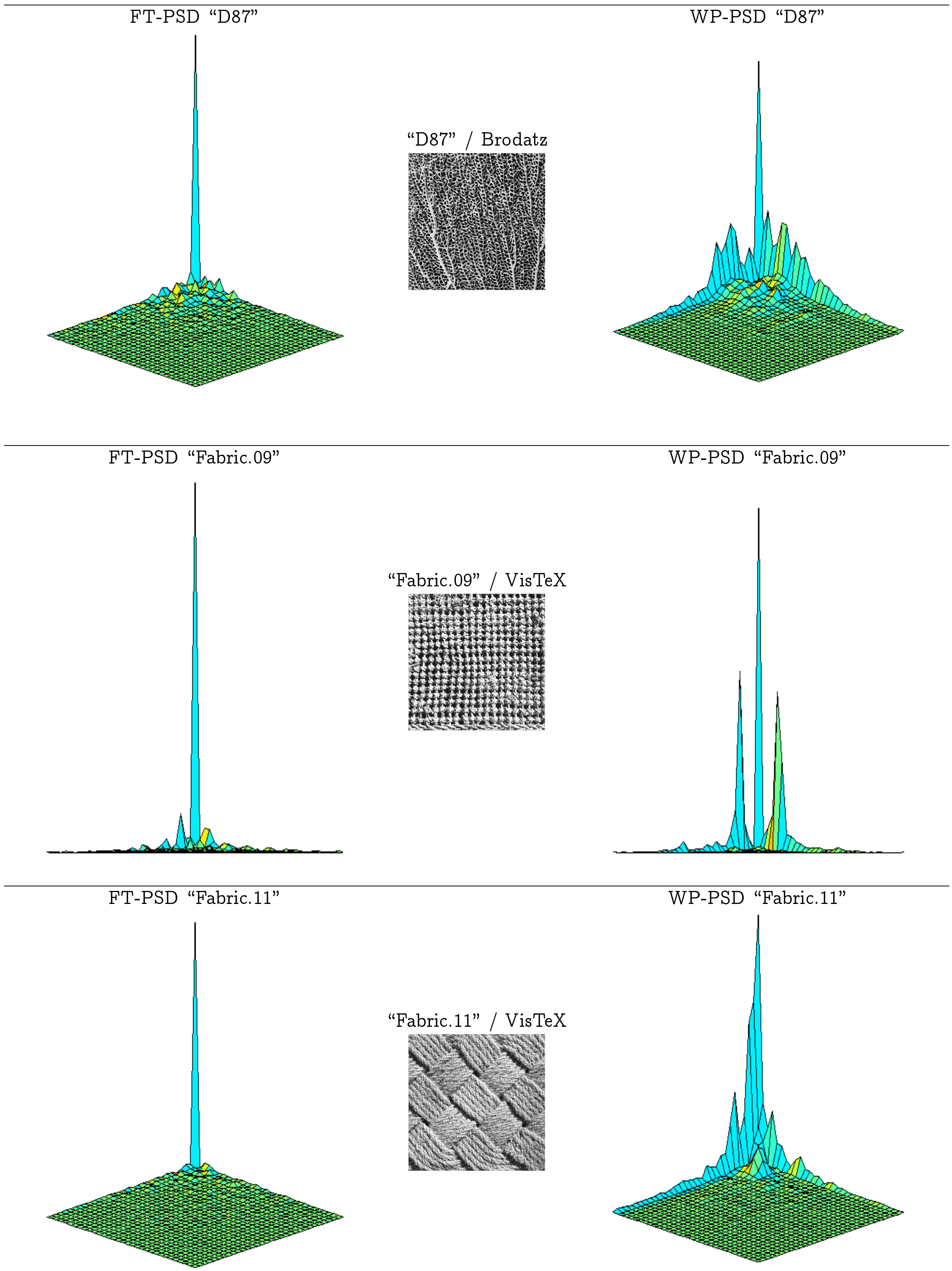

Fig. 2. Texture images and their spectra $\widehat{\gamma}$ computed by using Fourier and wavelet packets. Abscissa of the spectra images consist of a regular grid over $[0, \pi/2] \times [0, \pi/2]$.